647510 **MSC2MatE201x** Seat No :________

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
April/May-2023 (NEW COURSE)
Classical Mechanics -2 (Elective)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer the following questions. (Any Two) (10)

1. Show that the total angular momentum about a point is the angular momentum of motion concentrated at the center of mass, plus the angular momentum about the center of mass.
2. Derive the expression of moment of inertia about the axis of rotation.
3. Discuss in details the motion of a heavy symmetric top and derive analytic solution for the same.

Q.1(B) Answer the following question. (Any One) (04)

1. Give two examples for conservative of angular momentum.
2. Discuss one example of moment of inertia and heavy symmetric top.

Q.2(A) Answer the following questions. (Any Two) (10)

1. Derive inverse special Lorentz transformation.
2. Show that wave equation is invariant under special Lorentz transformation.
3. Prove that motion under the constant force as hyperbolic motion.

Q.2(B) Answer the following question. (Any One) (04)

1. Let the velocity of particle relative to frame S be $\bar{u}$. Find the $\bar{u}'$ relative to another inertial frame S' moving with speed v along x-axis relative to S.
2. Find the expression of force in relativistic mechanics.

Q.3(A) Answer the following questions. (Any Two) (10)

1. Derive Hamilton's equation of motion.
2. Obtain Routhian equations of motion for a system with Lagrangian
   $L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r}$, where r, θ are generalized coordinates.
3. **(I)** Show that a coordinate q_j is cyclic in L then it is cyclic in H.

   (II) If H is Hamiltonian of a system, then show that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$.

Q.3(B) Answer the following question. (Any One) (04)

1. State and prove the principle of least action.
2. Compute Routhian for the Hamiltonian $H = \frac{p^2}{4} + 2q^2$.

Q.4(A) Answer the following questions. (Any Two) (10)

1. Show that the transformation $Q = \log\left(\frac{\sin p}{q}\right); P = q\ cot\ p$ is canonical using fundamental Poisson brackets.
2. Show that symplectic condition is satisfied for an infinitesimal canonical transformation.
3. Derive Hamilton-Jacobi equation for Hamilton's principal function.

Q.4(B) Answer the following question. (Any One) (04)

1. Derive canonical transformation generated by the function $F_1(q, Q, t)$.
2. Show that **(I)** Poisson bracket is bilinear **(II)** Poisson bracket is anti-symmetric.

Q.5(A) Answer the following questions. (Any Two) (10)

1. **(I)** Define Hamilton's principle and principle of least action.

   (II) Derive the transformation equation of Newtonian relativity.
2. Find the Hamiltonian and Hamilton's equation of motion for the Lagrangian $L = \dot{q}_1^2 + \frac{\dot{q}_1^2}{a+bq_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2$, where q_1, q_2 are generalized coordinates.
3. Obtain Euler's equations of motion for a rigid body with one point fixed.

Q.5(B) Answer the following question. (Any One) (04)

1. State and prove Poisson's theorem.
2. Discuss the matrix form of special Lorentz transformation.

****